On the State Space of Fuzzy Cognitive Maps using Shrinking Functions

{ Leonardo Concepción, Gonzalo Nápoles, Rafael Bello and Koen Vanhoof }

Objectives

This study is devoted to understanding the foundations behind **Fuzzy Cognitive Maps (FCMs)**. Our main goals are:

1. to introduce several definitions and theorems that unveil the dynamic behavior of FCM-based models equipped with transfer F-functions
2. to estimate bounds for the activation value of each neuron and analyzing the covering and proximity of feasible activation spaces
3. to prove the convergence of the state space of any FCM model equipped with transfer F-functions
4. to understand the poor performance of FCMs facing complex simulation problems

Introduction

FCMs [1] are recurrent neural networks for modeling complex systems and the existing theoretical studies are mainly devoted to convergence issues [2].
Concerning the theoretical analysis of FCMs' dynamics, we summarize our paper **Unveiling the Dynamic Behavior of Fuzzy Cognitive Maps** [3]. First, we introduce definitions and theorems that allow studying the dynamic behavior of FCMs. The strong version of our theorem proves that the state space of an FCM shrinks infinitely and converges to a so-called **limit state space**. As a second contribution, we explore the covering and proximity of *feasible activation spaces*, which help explain why we should not expect impressive prediction rates when the model has low covering values as the FCM feasible state space is small.

Shrink Functions and State Space Estimation

We define F as the set of all monotonically increasing functions bounded into non-negative intervals. Also, let $f_i \in F$ be the transfer function used in the activation process of neuron C_i in the FCM. In [3], we refer to an F-function as any function belonging to F.
Let $\mathcal{H}_W$ and $\mathcal{H}_T$ be functions that take an FCM-based model $\mathcal{M}$ and a feasible state space at the t-th iteration $\mathcal{S}^{(t)}$ for this map and return a feasible state space at the $(t+1)$-th iteration $\mathcal{S}^{(t+1)}$ for the same map. While $\mathcal{H}_W$ uses the weight matrix W of $\mathcal{M}$ to calculate a feasible state space for the $(t+1)$-th iteration, $\mathcal{H}_T$ uses the FCM's topology only. Based upon estimated bounds for the successive activation values and from the monotonically increasing property of $f_i \in F$, we assert that over the same FCM, these two shrink functions transform feasible state spaces into state spaces which are also feasible.

To show that FCMs are not completely unpredictable, we propose two theorems as the pillars of our state-space estimation: the **Weak Shrinking State Space (WSSS)** and the **Strong Shrinking State Space (SSSS)**. The former asserts that the state spaces shrink from one iteration to the next one, although it is possible that $\mathcal{S}^{(t)} = \mathcal{S}^{(t+1)}$, which would imply that $\mathcal{S}^{(t)} = \mathcal{S}^{(t+k)}\ \forall k \in \mathbb{N}$. So, the state spaces may not shrink forever. The latter only varies in the sense that transfer functions are now bounded into open intervals. This means that the state space bounds are never reachable and hence, the state spaces will shrink forever and they will have a limit. The **limit state space** of $\mathcal{M}$ is $\mathcal{S}^{(\infty)} = \lim_{t\to\infty} \mathcal{S}^{(t)}$, when state spaces are iteratively calculated using either shrink function $\mathcal{H}_T$ or $\mathcal{H}_W$. According to simulations, $\mathcal{S}^{(\infty)}$ often contains a single point.

Covering and Proximity of FCM Models

Here, we discuss two proposed evaluation measures that help understand the properties of FCM-based systems. The **covering** quantifies the proportion of the induced activation space that is reachable by the neuron's activation values and the **proximity** measures the mean relative distance of neuron's activation values to the feasible activation spaces.
The results confirmed that better predictions for FCMs' behavior arise working with stable maps and their weight sets. Small covering values are evidence of the reduced representativeness of induced activation space, but sometimes we desire high covering values to represent the most diverse sets of outputs. As illustrated, such measures have a straightforward connection with the **SSSS Theorem**. More importantly, they help explain why FCMs sometimes perform poorly when applied to prediction problems that demand high accuracy.

Simulation results

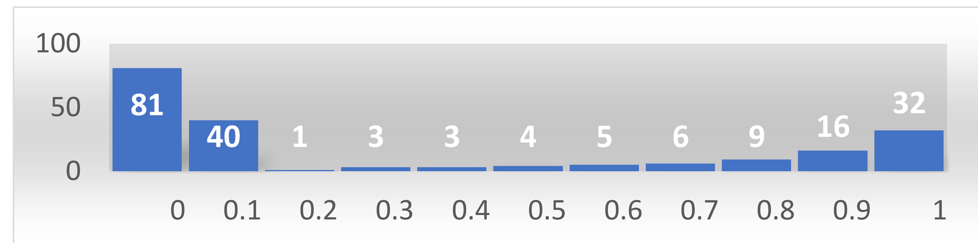

Figure 1: Distribution of covering values for stable FCMs using the $\mathcal{H}_W$ shrink function.

The simulations reported more valuable results in the presence of stable FCM models and when the knowledge comprised into the weight set is available. This scenario has proven useful to predict the FCMs' behavior (e.g., fixed-point attractors) by means of covering and proximity values. This allows envisaging, to some extent, the FCM model's behavior before the inference stage.

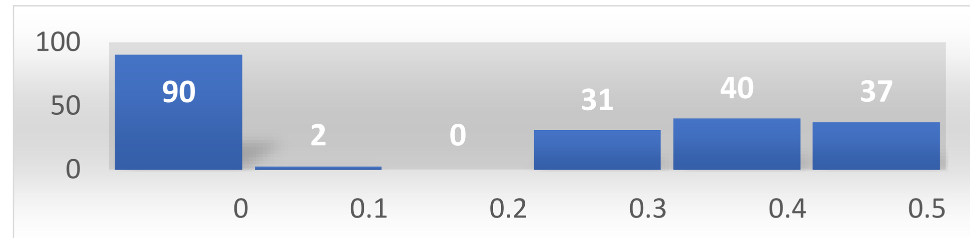

Figure 2: Distribution of proximity values for stable FCMs using the $\mathcal{H}_W$ shrink function.

References

[1] Bart Kosko. Fuzzy cognitive maps. *International Journal Man-Machine Studies*, 24(1):65–75, 1986.

[2] Gonzalo Nápoles, Leonardo Concepción, Rafael Falcon, Rafael Bello, and Koen Vanhoof. On the accuracy-convergence tradeoff in sigmoid fuzzy cognitive maps. *IEEE Transactions on Fuzzy Systems*, 26(4):2479–2484, 2018.

[3] L. Concepción, G. Nápoles, R. Falcon, K. Vanhoof, and R. Bello. Unveiling the dynamic behavior of fuzzy cognitive maps. *IEEE Transactions on Fuzzy Systems*, 2020.

Concluding remarks

In [3], we have introduced a theoretical formalism consisting of definitions and theorems to unveil the dynamical behavior of FCMs equipped with transfer F-functions, from the perspective of their state spaces.
The **SSSS Theorem** enunciated in this paper ensures that the feasible state space of the targeted FCMs shrinks infinitely, yet the system converges to its **limit state space**. As shown in the experiments, approximating an FCM's limit state space is useful to predict fixed-point attractors. Likewise, we illustrated that the covering of feasible activation spaces is often poor and irregular for FCMs with reduced network topologies.
As future research, this knowledge could be injected into the learning procedure in order to improve network's performance.

Full paper & Contact Info

lcperez@uclv.cu
G.R.Napoles@tilburguniversity.edu