# Towards Partial Order Reductions for Strategic Ability


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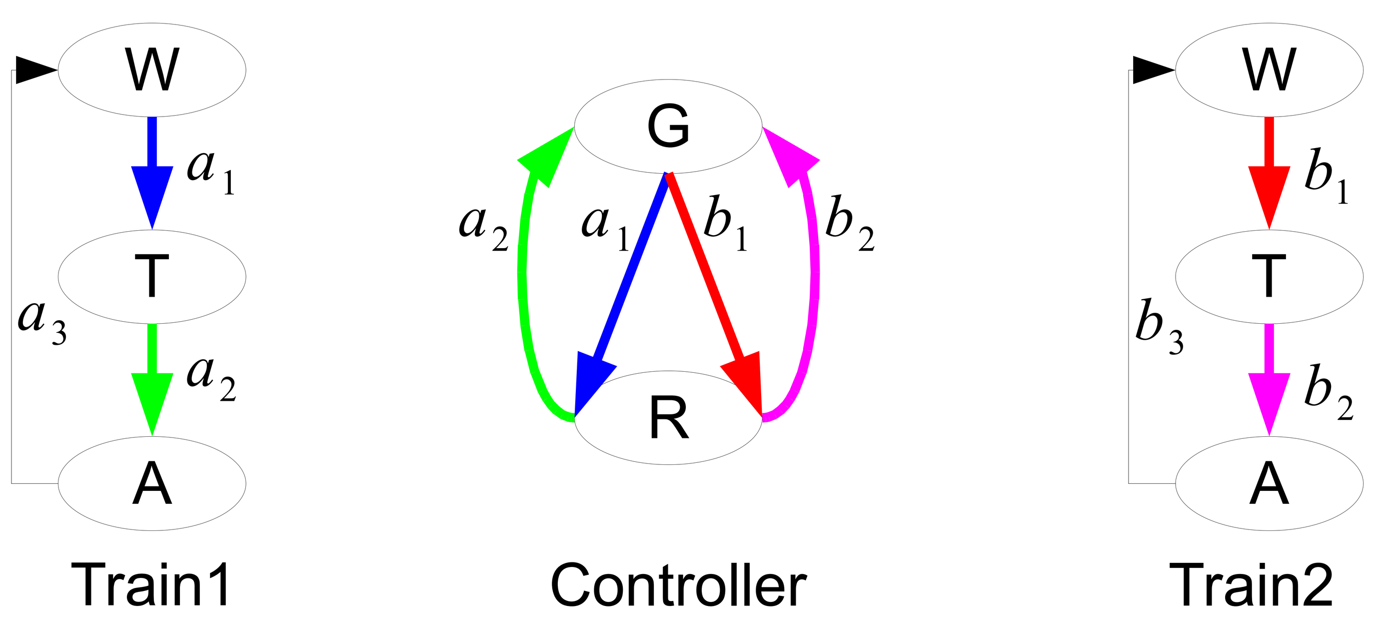
## Contribution in a Nutshell 1

- We propose a general semantics for **strategic abilities** in **asynchronous systems**, with and without perfect information
- We establish the complexity of model checking $\leadsto$ no surprises here
- Most importantly, we develop a methodology for **partial order reduction (POR)** in verification of abilities under imperfect information
- The methodology is **as powerful as the reductions for linear time logic**
- Interestingly, it does not work for abilities under perfect information

## Strategic Logics 2

- **Strategic logics** provide powerful tools to reason about multi-agent systems
- We focus on a variant of **ATL*** with imperfect information and memoryless strategies, called **sATL$^*_{ir}$**
- Important for specification and verification of functionality and security requirements
- Example formulae:
  - $\langle\langle alice \rangle\rangle \mathrm{F\,win}$: Alice can eventually win no matter what the other agents do
  - $\langle\langle alice, bob \rangle\rangle \mathrm{G\,safe}$: Alice and Bob together can guarantee that the system will always remain in a safe state
  - $\neg\langle\langle tr_1, tr_2 \rangle\rangle \mathrm{F}(\mathrm{in}_1 \vee \mathrm{in}_2)$: trains 1 and 2 cannot enter the tunnel on their own, even if they collaborate




Asynchronous MAS: 2 trains and a controller

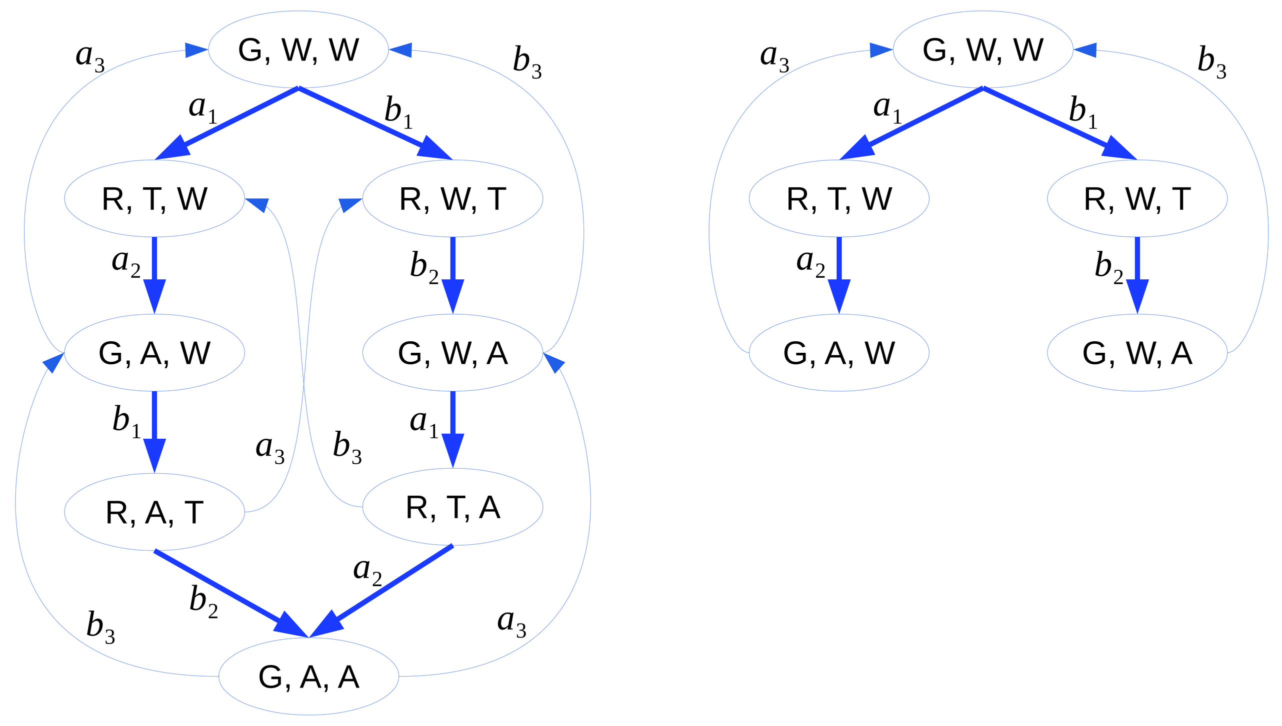
## Asynchronous Multi-Agent Systems 3

- The semantics of strategic logics are almost exclusively based on synchronous concurrent game models
- However, many multi-agent systems are inherently **asynchronous** or have simple asynchronous abstractions
  $\leadsto$ need to adapt the semantics of strategic ability
- Moreover, asynchronous models suffer from **state-state space explosion** due to interleaving...
- ...but **effective reduction methods** exist for linear time temporal properties in asynchronous distributed systems

*Question: Can we adapt the methods to the more expressive language?*

## Partial Order Reductions for LTL and sATL$^*_{ir}$ 4

- A stack represents the visited path $\pi = g_0 a_0 g_1 a_1 \cdots g_n$ of $M'$.
- For $g_n$, the following three operations are computed in a loop:
- The set $en(g_n) \subseteq Act$ of enabled actions is identified and a subset $E(g_n) \subseteq en(g_n)$ of necessary actions is heuristically selected.
- For any action $a \in E(g_n)$ compute the successor state $g'$ of $g_n$ such that $g_n \xrightarrow{a} g'$, and add $g'$ to the stack.
  Recursively proceed to explore the submodel originating at $g'$
- Remove $g_n$ from the stack.




Unfolding the trains and controller system: full model $M$ (left), reduced model $M'$ (right). Visible transitions are depicted by blue bold arrows

## Conditions for selection of $E(g)$ for LTL 5

**C1** Along each path $\pi$ in $M$ that starts at $g$, each action $a \in Act \setminus E(g)$ that is dependent on an action in $E(g)$ cannot be executed in $\pi$ without an action in $E(g)$ is executed first.

**C2** If $E(g) \neq en(g)$, then each action in $E(g)$ is invisible, i.e., does not change valuation of $g$.

**C3** For every cycle in $M'$ there is at least one node $g$ in that cycle for which $E(g) = en(g)$.

## Theorem (1)

*Partial order reductions under C1 and C3 preserve sATL$^*_{ir}$ under concurrency-fairness.*

Assume that the actions of the agents in $A \subset Agents$ are visible.

## Theorem (2)

*Partial order reductions under C1, C2, C3 preserve the formulas of sATL$^*_{ir}$ that refer only to coalitions $A' \subseteq A$.*

## Takeaway Message 6

Takeaway message: **free lunch is sometimes possible** ☺

Main contribution: **proving** that the powerful variant of POR for LTL can be applied to **much more expressive language of sATL$^*_{ir}$** (nontrivial!)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