Towards Partial Order Reductions for Strategic Ability

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Contribution in a Nutshell

- We propose a general semantics for strategic abilities in asynchronous systems, with and without perfect information.
- We establish the complexity of model checking $\sim$ no surprises here.
- Most importantly, we develop a methodology for partial order reduction (POR) in verification of abilities under imperfect information.
- The methodology is as powerful as the reductions for linear time logic.
- Interestingly, it does not work for abilities under perfect information.

Strategic Logics

- Strategic logics provide powerful tools to reason about multi-agent systems.
- We focus on a variant of ATL with imperfect information and memoryless strategies, called $\mathtt{sATL}_\omega$.
- Important for specification and verification of functionality and security requirements.
- Example formulae:
  - $(\\langle \text{alice} \rangle \square \text{win})$: Alice can eventually win no matter what the other agents do.
  - $(\\langle \text{alice}, \text{bob} \rangle \square \text{safe})$: Alice and Bob together can guarantee that the system will always remain in a safe state.
  - $\neg(\langle \text{train}_1, \text{train}_2 \rangle \square (\text{in}_1 \lor \text{in}_2))$: trains 1 and 2 cannot enter the tunnel on their own, even if they collaborate.

Asynchronous Multi-Agent Systems

- The semantics of strategic logics are almost exclusively based on synchronous concurrent game models.
- However, many multi-agent systems are inherently asynchronous or have simple asynchronous abstractions $\sim$ need to adapt the semantics of strategic ability.
- Moreover, asynchronous models suffer from state-state space explosion due to interleaving…
- …but effective reduction methods exist for linear time temporal properties in asynchronous distributed systems.

Question: Can we adapt the methods to the more expressive language?

Partial Order Reductions for LTL and $\mathtt{sATL}_\omega$

- A stack represents the visited path $\pi = g_0a_0g_1a_1 \cdots g_n$ of $M'$.
- For $g_n$, the following three operations are computed in a loop:
  - The set $\text{en}(g_n) \subseteq \text{Act}$ of enabled actions is identified and a subset $\text{Act}(g_n) \subseteq \text{en}(g_n)$ of necessary actions is heuristically selected.
  - For any action $a \in \text{Act}(g_n)$ compute the successor state $g'$ of $g_n$ such that $g_n \xrightarrow{a} g'$, and add $g'$ to the stack.
  - Recursively proceed to explore the submodel originating at $g'$.
- Remove $g_n$ from the stack.

Conditions for selection of $E(g)$ for LTL

C1 Along each path $\pi$ in $M$ that starts at $g$, each action $a \in \text{Act} \setminus E(g)$ that is dependent on an action in $E(g)$ cannot be executed in $\pi$ without an action in $E(g)$ is executed first.

C2 If $E(g) \neq \text{en}(g)$, then each action in $E(g)$ is invisible, i.e., does not change valuation of $g$.

C3 For every cycle in $M'$ there is at least one node $g$ in that cycle for which $E(g) = \text{en}(g)$.

Theorem (1)

Partial order reductions under C1 and C3 preserve $\mathtt{sATL}_\omega$ under concurrency-fairness.

Assume that the actions of the agents in $A \subseteq \text{Agents}$ are visible.

Theorem (2)

Partial order reductions under $C1$, $C2$, $C3$ preserve the formulas of $\mathtt{sATL}_\omega$ that refer only to coalitions $A' \subseteq A$.

Takeaway Message

Takeaway message: free lunch is sometimes possible 😊

Main contribution: proving that the powerful variant of POR for LTL can be applied to much more expressive language of $\mathtt{sATL}_\omega$ (nontrivial!)